Control of laser wake field acceleration by plasma density profile

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We show that both the maximum energy gain and the accelerated beam quality can be efficiently controlled by the plasma density profile. Choosing a proper density gradient one can uplift the dephasing limitation. When a periodic wake field is exploited, the phase synchronism between the bunch of relativistic particles and the plasma wave can be maintained over extended distances due to the plasma density gradient. Putting electrons into the $n-$th wake period behind the driving laser pulse, the maximum energy gain is increased by the factor $2\pi n$ over that in the case of uniform plasma. The acceleration is limited then by laser depletion rather than by dephasing. Further, we show that the natural energy spread of the particle bunch acquired at the acceleration stage can be effectively removed by a matched deceleration stage, where a larger plasma density is used.

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Plasma-based schemes of electron acceleration have recently demonstrated impressive progress. Quasimonoenergetic electron bunches with the energy up to 1 GeV and with the charge of 50 pC have been generated in experiments [1]. All-optical methods for control of the bunch parameters have been developed also [2]. It is generally believed that electrons in these experiments have been accelerated in the "Bubble regime" [3]. At the same time, a significant advance in plasma profile engineering has been achieved to make laser plasma interaction more efficient: plasma capillaries for laser guiding [4], fabrication of corrugated plasma structures [5], plasma machining [6].

One of the main limitations on energy gain in laser-plasma accelerators comes from the dephasing. The velocity of relativistic electrons is slightly higher than the phase velocity of the wake, which is determined by the group velocity of the driving laser pulse. The accelerated electrons slowly outrun the plasma wave and leave the accelerating phase.

The limitation caused by the dephasing can be overcome by employing a proper plasma gradient [7]. The profile of the plasma density should be such that the advance of the accelerated electrons matches the advance of the plasma wave. The equation for plasma density profile in the 1D configuration is

$$\frac{d}{dx}\left(\frac{\Phi_n}{\omega_p(x)}\right) \simeq 1 - \frac{c}{v_{gr}}, \tag{1}$$

where $\omega_p^2(x) = 4\pi e^2 n(x)/m$ is the squared plasma frequency, $n(x)$ is the plasma density, $\Phi_n = \text{const}$ is the phase of ultrarelativistic electrons trapped n plasma wavelengths ($\lambda_p = 2\pi c/\omega_p$) behind the laser pulse, v_{gr} is the group velocity of the laser pulse, c is the speed of light, e and m are the electron charge and mass respectively. For weakly relativistic laser pulses with $a = eA/mc^2 \ll 1$ and for rarefied plasmas $n/n_c \ll 1$ we can assume $v_{gr}/c \simeq 1 - n(x)/2n_c$ and $\gamma \gg \gamma_{gr}$, where γ is the relativistic gamma-factor of the accelerated electrons, $\gamma_{gr}^2 = 1 - v_{gr}^2/c^2$, $n_c = m\omega^2/4\pi e^2$ is the critical plasma density and ω is the laser frequency. The solution of Eq. (1) for the phase synchronism in laser wake field acceleration is

$$n(x) = \frac{n_0}{(1 - x/L_{inh})^{2/3}}, \tag{2}$$

$$L_{inh} = \frac{c}{\omega}\left(\frac{n_0}{n_c}\right)^{-3/2}\frac{2\Phi_n}{3}, \tag{3}$$

where $n_0 = n(x=0)$. It follows from Eq. (2) that the plasma density increases along the pulse propagation and the acceleration distance is limited by L_{inh} since the plasma density goes to infinity at $x = L_{inh}$.

Consider a short circularly polarized laser pulse with the Gaussian envelope $a^2(\xi) = a_0^2 \exp\left(-\tau^2/T^2\right)$, where $\tau = t - \int^x dx'/v_{gr}(x')$. For simplicity, the ion dynamics, the thermal motion and the transverse dynamics of plasma electrons are neglected. It is also assumed that the accelerated electrons do not affect plasma wake structure. The accelerating force on the relativistic electrons in the plasma wake can be presented in the form [8] $F_x = -\left(\sqrt{\pi}/2\right) a_0^2 mc\omega_p^2(x) T \exp\left[-\omega_p^2(x)T^2/4\right]\cos\Phi(x)$, where $\Phi(x) = \omega_p(x)\tau(x) \simeq \sqrt{n(\xi)/n_c}\int n(\xi)/\left(2n_c\right) d\xi$ and $\xi = \omega x/c$. For $\cos\Phi = -1$ the accelerating force achieves a maximum value of $F_x \simeq mc\omega_p\sqrt{\pi/2}a_0^2\exp(-1/2)$ when $T = \sqrt{2}/\omega_p$. Therefore for the given duration of the laser pulse there is the optimal density ($\omega_p^2 T^2 = 2$), at which the accelerating force peaks. When the laser pulse propagates in inhomogeneous plasma the pulse duration will be soon out of optimal value. In addition, the nonlinear dynamics of the

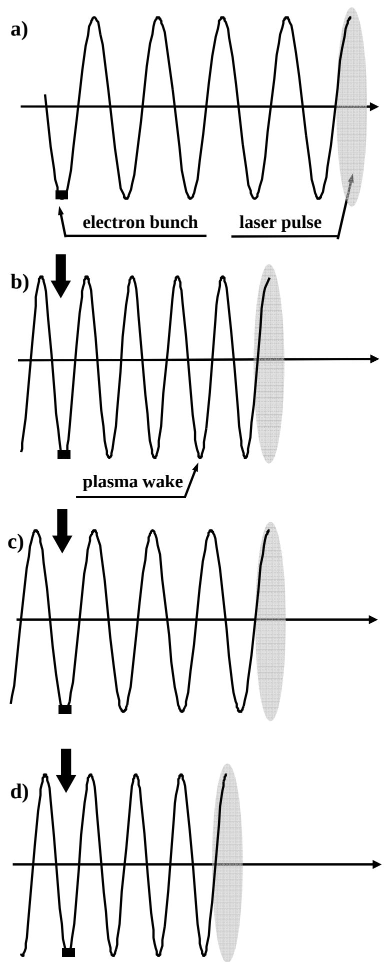

FIG. 1: Electron acceleration in plasma layers in the frame of light speed (schematically). The ultra-relativistic electron bunch (black rectangle) is always located at the peak value of the accelerating field.

laser pulse during propagation can significantly modify the pulse envelope and carrier frequency. The both effects influence the energy gain and should be taken into account to find the plasma density profile for optimal acceleration.

The electron energy gain is $\Delta\mathcal{E} = \left(\int F_x dx\right)$. Introducing the parameter $\mu = \int (n/n_c)\, dx$ the phase can be rewritten in the form $\Phi = \mu\sqrt{\dot\mu}/2$ and the energy gain can be presented as follows

$$\Delta\mathcal{E} = \int F(\xi,\mu,\dot\mu)d\xi, \tag{4}$$

$$F(\xi,\mu,\dot\mu) = \frac{\sqrt{\pi}}{2}a_0^2\dot\mu\beta(\xi)\exp\left[-\frac{\beta^2(\xi)\dot\mu}{4}\right]\cos\left(\frac{\mu\sqrt{\dot\mu}}{2}\right) \tag{5}$$

where the evolution of the pulse duration is assumed to be known $\beta(\xi) = \omega T(\xi)$. Considering $\Delta\mathcal{E}\left[\mu\left(\xi\right)\right]$ as a functional the Euler-Lagrange equation can be derived for $\mu\left(\xi\right)$ providing the peak gain

$$\frac{\partial F}{\partial \mu} - \frac{d}{d\xi}\frac{\partial F}{\partial \dot\mu} = 0. \tag{6}$$

Assuming that the laser pulse is short, $\omega_p T \ll 1$, so that the pulse duration effects can be neglected $\beta(\xi) = 0$, Eq. (6) reduces to Eq. (1) with $\Phi_n = -\pi(1+2n)$, where $n = 1, 2, ...$

Integrating Eq. (1) for homogeneous plasma ($n = n_0 =$ const) the wake phase at the bunch position can be calculated $\Phi = -3\pi/2 + (n_0/n_c)^{3/2}\xi/2$, where it is assumed that $\Phi(\xi = 0) = -3\pi/2$. The electron acceleration is possible in the range $0 < x < L_{\mathrm{hom}}$, when $-\pi(3/2+2n) < \Phi < -\pi(1/2+2n)$, where $n = 1, 2, ...$ and $L_{\mathrm{hom}} = 2\pi c(n_0/n_c)^{-3/2}/\omega$ is the well known expression for detuning length in homogeneous plasma [8]. Integrating Eq. (4) we obtain the known expression for the peak gain in the electron energy for homogeneous plasma [9] $\Delta\mathcal{E}_{\mathrm{hom}} = 2\sqrt{2\pi}\exp(-1/2)a_0^2(n_c/n_0)mc^2$, where optimal duration of the laser pulse $T = \sqrt{2}/\omega_p$ was assumed.

For a short laser pulse $\omega_p T \ll 1$ the peak gain can be achieved when the wake phase at the electron position is $\Phi_n = -\pi(1+2n)$ and the plasma density profile obeys Eq. (2). Integrating Eq. (4) the energy gain takes the form

$$\Delta\mathcal{E}_{inh}(x) \simeq \mathcal{E}_0\left[1 - \sqrt{\frac{n_0}{n(x)}}\right], \tag{7}$$

where $\mathcal{E}_0 = (3/2)\sqrt{\pi}a_0^2mc^2(n_0/n_c)\,(\omega T)\,(\omega L_{inh}/c)$. The energy gain over the distance $0 < x < L_{inh}$ is $\Delta\mathcal{E}_{inh} \simeq \mathcal{E}_0 \simeq \Delta\mathcal{E}_{\mathrm{hom}}2^{-3/2}\exp(1/2)\omega_{p0}T\Phi_n$, where $\omega_{p0}^2 = 4\pi e^2 n_0/m$. It follows from the obtained expression that the gain increases as Φ_n. Therefore, the electron acceleration is most efficient when the electrons are loaded at the peak accelerating field as far behind the laser pulse as possible. For arbitrary values of $\omega_p T$, and the phase synchronism $\Phi_n =$ const ensured by the plasma profile (2), the energy gain is

$$\Delta\mathcal{E}_{inh}\left(x\right) = \mathcal{E}_0\left[\Psi(x) - \Psi(0)\right],$$

$$\Psi(x) = \sqrt{\frac{\delta(0)}{\delta(x)}}\exp\left[\delta(x)\right] + \sqrt{\pi\delta(0)}\mathrm{erf}\left[\sqrt{\delta(x)}\right] \tag{8}$$

where $\delta(x) = \omega_p^2(x)T^2/4$ and $\delta(0) = \omega_{p0}^2T^2/4 = n_0\omega^2T^2/4n_c$. In the limit $\omega_p T \ll 1$ ($\delta \ll 1$) Eq. (8) is reduced to Eq. (7).

Keeping the bunch always in the same wave period behind the laser pulse may become unpractical, because the plasma density would vary too strongly over the full acceleration stage. To optimize the process we propose a layered plasma density profile. The acceleration scheme is illustrated in Fig. 1. Let the electrons be trapped in the n-th plasma period during acceleration in the first

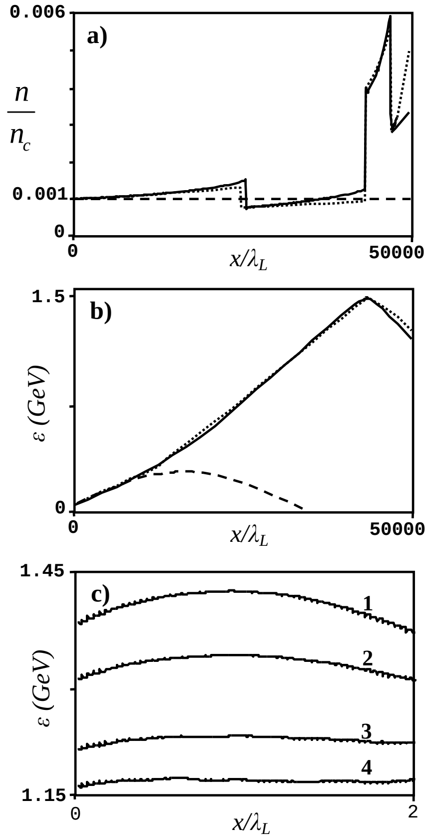

FIG. 2: Electron bunch acceleration and energy spread reduction in plasma layers: (a) the plasma density profile, (b) the mean energy of the electron bunch (c) electron energy vs electron position in the bunch. Electrons are accelerated by the first laser pulse in the first two layers ($0 < x < 43370\lambda_L$) whereas they are decelerated in the third and fourth layers ($43370\lambda_L < x < 49820\lambda_L$). The solid line and the dotted line correspond to the PIC simulation results and the theoretical estimates, respectively. To compare, the dashed line shows acceleration in homogeneous plasma with the constant density $n_0 = 0.001 n_c$. The energy distributions in frame (c) are shown at the beginning of the deceleration at $x = 43370\ \lambda_L$ (line 1), at $x = 45370\ \lambda_L$ (line 2), at $x = 47370\ \lambda_L$ (line 3) and the end of deceleration at $x = 49820\ \lambda_L$ (line 4), respectively. The laser pulses are circularly polarized with Gaussian profile and wavelength 1 μm. The laser pulse parameters are $T = 10.6\lambda_L/c$, $a_0 = 0.6$ for the pulse on the acceleration stage and $T = 5.3\lambda_L/c$, $a_0 = 0.5$ for the pulse on the deceleration stage.

plasma layer where the plasma density increases from n_1 to some final value n_f in accordance with Eq. (2). The wake phase is synchronized to the electron bunch in this plasma layer due to the plasma density gradient. The wake phase at the electron bunch center is $\Phi_n = \pi(2n-1)$ that corresponds to the maximum of the accelerating force. When the plasma density reaches the value of n_f, the electron bunch enters the second plasma layer where the plasma density increases from n_2 to n_f. The density value of $n_2 = n_f \Phi_{n-1}^2/\Phi_n^2$ is chosen such that the electrons in the second layer become located in the $(n-1)$-th plasma period behind the laser pulse. The wake phase at the electron bunch center is $\Phi_{n-1} = \pi(2n-3)$ that again corresponds to the maximum of the accelerating force. Inductively, the plasma density increases from $n_m = n_f \Phi_{n-m}^2/\Phi_{n-m-1}^2$ to n_f in the m-th plasma layer in accordance with Eq. (2) and the electrons are trapped in the $(n-m)$-th plasma period behind the laser pulse. The successive acceleration in the several plasma layers leads to a significant increase in the electron energy gain and can be accomplished by *the same driving laser pulse* until it is depleted. The transition between the plasma density layers can be smooth, but much shorter than the length of the layers themselves.

Further, the proposed scheme allows us to control the energy spectrum of the bunch electrons. Naturally, electrons located at the bunch center get the greatest energy gain while electrons located at the bunch edges get the smallest energy gain. This leads to an energy spread of the bunch after the acceleration stage. To reduce this spread we suggest to use a matched decelerating stage with a higher plasma density. Although the decelerating stage will reduce the net energy gain, its effect on the energy spread reduction is much stronger, because the plasma wake period decreases as the plasma density increases.

To get an estimate of the energy spread reduction we assume that the electron bunch duration is much shorter than the plasma wave period while the plasma density profile and energy gain are described by Eqs. (2) and (7), respectively. It is also assumed that the bunch center is always synchronized to the maximum of accelerating or decelerating forces $\Phi_n = -\pi n$, $n = 1, 2, ...$ Expressing the phase Φ_n in $\omega_p \eta/c \ll 1$, where η is the distance from the electron to the bunch center, the force acting at the electron located at the position η can be Taylor expanded as $F_x = F_{x0}(1 - (\omega_p \eta/c)^2/2)$, where F_{x0} is the force on the electron in the bunch center. The difference in the accelerating force acting on electrons located at different positions in the bunch leads to the energy spread of the bunch electrons. Using Eq. (7) the energy gain as a function of the electron position in the bunch is

$$\Delta\mathcal{E}(x,\eta) = \Delta\mathcal{E}_{inh}(x) - \alpha(x)\eta^2 \qquad (9)$$

$$\alpha(x) = \mathcal{E}_0 \frac{\omega_{p0}^2}{2c^2}\left[1 - \sqrt{\frac{n}{n_0}}\right], \qquad (10)$$

where $\Delta\mathcal{E}_{inh}$ is defined by Eq. (7). Introducing the energy spread as $\sigma = \left(\langle\mathcal{E}^2\rangle - \langle\mathcal{E}\rangle^2\right)^{1/2}$, where $\langle\mathcal{E}\rangle = \int_{l_b/2}^{l_b/2}(\mathcal{E}/l_b)d\eta$, $\langle\mathcal{E}^2\rangle = \int_{l_b/2}^{l_b/2}(\mathcal{E}^2/l_b)d\eta$ and l_b/c is the bunch duration, we find the evolution of the bunch energy spread

$$\sigma(x) = \frac{\alpha(x) l_b^2}{6\sqrt{5}} = \mathcal{E}_0 \left[\sqrt{\frac{n(x)}{n_0}} - 1 \right] \frac{\omega_{p0}^2 l_b^2}{12\sqrt{5} c^2}. \qquad (11)$$

Let the bunch be accelerated in the first plasma layer where the plasma density increases from n_1 to $4n_1$. Then, it is decelerated in the second layer where the density increases from n_2 to $4n_2$. The parameters of the laser pulse are assumed to be the same in the both layers. The total energy gain after passing the two plasma layers is $\mathcal{E} = \mathcal{E}_1[1 - (\Phi_2/\Phi_1)(n_1/n_2)^{1/2}]$ while the total spread is $\sigma = \sigma_1|1 - (\Phi_2/\Phi_1)(n_2/n_1)^{1/2}|$, where $\mathcal{E}_1$ and σ_1 are the energy gain and spread after passing the first layer, respectively, Φ_1 and Φ_2 are the phase of the bunch center in the first and second layers, respectively. Choosing $n_2/n_1 = \Phi_1^2/\Phi_2^2$ the final spread is removed completely in this approximation. At the same time, the energy gain is $\mathcal{E} = \mathcal{E}_1(1 - n_1/n_2)$. For $n_2/n_1 \simeq 4$ the bunch loses only one quarter of its energy after passing the decelerating layer while the energy spread will be completely removed.

In order to check the validity of our simplified model, we have performed 1D PIC simulations, using the code Virtual Laser Plasma Laboratory [10]. The code has been supplemented with adaptive scheme: the plasma density is varied in each time step so that the bunch center is always located in the maximal value of the accelerating force. The plasma wake excited by the first laser pulse in the first two layers accelerates the electron bunches while the bunch is decelerated in the second two layers where the plasma wake excited by the second pulse (see Fig. 2a). The laser pulses are circularly polarized with Gaussian profile. The laser wavelength $\lambda_L = 1$ μm.

The duration of the first pulse is $T = 10.6\lambda_L/c$ and $a_0 = 0.6$ while the duration of the second pulse is $T = 5.3\lambda_L/c$ and $a_0 = 0.5$. The plasma density at the beginning of acceleration is $n_0 = 0.001\, n_c$. The bunch center in the first layer is located at 3.5 plasma periods behind the pulse center (phase of bunch center is $\Phi_1 = -7\pi$). The second layer is chosen such that the bunch is located at 2.5 plasma periods behind the pulse center ($\Phi_2 = -5\pi$) there. For the decelerating layers $\Phi_3 = -8\pi$ and $\Phi_4 = -6\pi$ in the third and fourth layers. The electron bunch was initially monoenergetic with energy 50 MeV and duration $T_b = 2\lambda_L/c$.

The bunch energy achieves about 1.4 GeV with the energy spread of about 2% after acceleration in the first two plasma layers (see Fig. 2b,c). On the deceleration stage, the bunch energy reduces to 1.2 GeV whereas the energy spread reduces to less than 0.5% (see Fig. 2b,c). The total distance of the bunch acceleration and conditioning is about 5 cm. It is seen from Fig. 2a,b that the plasma density profile and the bunch energy evolution obtained in PIC simulation agree fairly good with the theoretical predictions (2) and (8). To achieve a better agreement we take into account the pulse compression in the second half of the first layer (we suppose that $a_0 = 0.7$ and $T = 7.8\lambda_L/c$), in the second layer ($a_0 = 0.8$ and $T = 5.9\lambda_L/c$) and in the fourth layer ($a_0 = 0.6$ and $T = 3.7\lambda_L/c$) in accordance with PIC simulation results. The deviation of the numerical results from theoretical estimates is caused by the complex nonlinear dynamics of the laser pulse during propagation. According to Fig. 2b, the energy gain in the inhomogeneous plasma is about 5 times more than that in the homogeneous plasma with $n = n_0 = \text{const}$.

The approach reported above is one-dimensional and thus does not take into account transverse dynamics of the laser pulse and electrons. The laser pulse can be efficiently guided by plasma channels over many Rayleigh lengths [4, 11]. More complicated can be the transverse dynamics of accelerated electrons. It is well known [9] that the simultaneous accelerating and focusing of electrons occurs over a quarter of the plasma wavelength. Another accelerating quarter of the plasma wave defocuses electrons. At the maximum of the accelerating fields, however, the transverse fields vanish. The use of a preformed plasma channel can significantly extend the region where the accelerating and focusing phases overlap [12].

In conclusion, we have proposed to control both the maximum energy gain and the accelerated beam quality by the plasma density profile. Choosing a proper density gradient one can uplift the dephasing limitation. The natural energy spread of the particle bunch acquired at the acceleration stage can be effectively removed by a matched deceleration stage, where a larger plasma density is used.

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